Q1. Sec 03. Grading On The Curve (GOTC), Inc. administers Operations Management tests to job candidates who are applying for CEO positions in leading Fortune 500 Companies. For this purpose, GOTC employs eight senior graders and ten junior graders (called in-house graders).

Each test consists of three parts: written, computer, and video-recorded. The time, in minutes, required by each kind of grader to grade each type of test is as follows:

<table>
<thead>
<tr>
<th>Grader</th>
<th>Written</th>
<th>Computer</th>
<th>Recorded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senior</td>
<td>20</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Junior</td>
<td>30</td>
<td>20</td>
<td>18</td>
</tr>
</tbody>
</table>

Each grader can work up to 40 hours in the coming week. 800 candidates have just taken the test, and the results have to be announced after a week. For quality control purposes, senior graders must grade at least 10% of all tests graded in-house in each category.

Work can also be assigned to outside part-time graders, who charge $25 for each written test, $15 for each computer test, and $20 for each recorded test, and can do any number of tests in a week. However, each test done by an outside grader requires a check by an in-house grader. A check by a senior grader takes 10 minutes, while a check by a junior grader takes 15 minutes.

Formulate an algebraic linear programming model to minimize the wages paid to outside graders, while still meeting the deadline for grading all the tests. Clearly define the decision variables, and explain the constraints.

Solution

Variables

For type of exam $X$ ($X = W, C, R$ for Written, Computer, and Recorded, respectively), and kind of grader $Y$ ($Y = S, J, P$ for Senior, Junior, and Part-time (outside) grader), denote by $XY$ the number of exams of type $X$ graded by graders of kind $Y$.

For example,

$RJ$ = the number of recorded exams graded by junior graders,

$CP$ = the number of computer exams graded by outside graders.

Also, $ES, EJ$ the number of exams checked by senior, and junior graders, respectively.

Objective

$$\text{min } 25WP + 15CP + 20RP$$

Constraints

$$WS + WJ + WP = 800 \quad (1)$$
$$CS + CJ + CP = 800 \quad (2)$$
$$RS + RJ + RP = 800 \quad (3)$$
$$WP + CP + RP = ES + EJ \quad (4)$$
$$\frac{20}{60} WS + \frac{15}{60} CS + \frac{15}{60} RS + \frac{10}{60} ES \leq 10 \cdot 40 \quad (5)$$
$$\frac{30}{60} WJ + \frac{20}{60} CJ + \frac{15}{60} RJ + \frac{15}{60} EJ \leq 10 \cdot 40 \quad (6)$$
$$WS \geq 0.1(WS + WJ) \quad (7)$$
$$CS \geq 0.1(CS + CJ) \quad (8)$$
$$RS \geq 0.1(RS + RJ) \quad (9)$$

All variable $\geq 0$

Q1. Sec 10. Manpower Planning. The Garden State Airline (GSA) must decide how many new cabin personnel (stewardesses & stewards) to hire and train over the next 6 months. The requirements are...
given in terms of flight-hours per month:

<table>
<thead>
<tr>
<th>Month</th>
<th>Hours Needed</th>
<th>Month</th>
<th>Hours Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>8,000</td>
<td>April</td>
<td>10,000</td>
</tr>
<tr>
<td>February</td>
<td>9,000</td>
<td>May</td>
<td>9,000</td>
</tr>
<tr>
<td>March</td>
<td>8,000</td>
<td>June</td>
<td>12,000</td>
</tr>
</tbody>
</table>

One month of training is required before a trainee becomes “experienced” (or a “regular crew member”). During the month of training, each trainee requires 100 hours of actual in-flight experience and supervision. Therefore, for each trainee, 100 less hours are available for flight service by regular personnel.

Each experienced crew member can work up to 150 hours in a month. The pay for experienced crew members is $3000/month, even if they work less than 150 hours. Experienced crew members may be asked to work up to 20 hours overtime, at the rate of $50/hour (in addition to their regular hours and pay.) The pay for trainees is $2000/month.

GSA has 60 regular crew members at the beginning of January. Each month afterwards, approximately 10% of the experienced personnel quit GSA on the first of the month.

Formulate the hiring-and-training problem as an LP model, where the objective is to minimize cost. Use $X_t$ for the number of new personnel beginning training in month $t$, ($t = 1, \cdots, 6$). Define any additional variables that you need.

**Solution**

**Variables**
For each month $t = 1, \cdots, 6$:

- $X_t$ the number of new personnel beginning training in month $t$,
- $Y_t$ the number of experienced personnel reporting to work at the beginning of month $t$,
- $O_t$ the number of hours worked overtime in month $t$.

**Objective**

\[
\min \left\{ 3,000 \sum_{t=1}^{6} Y_t + 2,000 \sum_{t=1}^{6} X_t + 50 \sum_{t=1}^{6} O_t \right\}
\]

**Constraints**

For each month $t = 1, \cdots, 6$

\[
150 Y_t + O_t - 100 X_t \geq \text{hours needed in month } t
\]

In particular

\[
150 Y_1 + O_1 - 100 X_1 \geq 8,000
\]

\[
\cdots \cdots
\]

\[
150 Y_6 + O_6 - 100 X_6 \geq 12,000
\]

For each month $t = 1, \cdots, 5$

\[
Y_{t+1} = 0.9Y_t + X_t, \ (t = 1, \cdots, 6) \quad \text{(personnel balance)} \quad (2)
\]

\[
Y_1 = 60 \quad \text{(initial personnel)} \quad (3)
\]

For each month $t = 1, \cdots, 6$

\[
O_t \leq 20 Y_t \quad \text{(limit on overtime)} \quad (4)
\]

\[
X_t, Y_t, O_t \geq 0, \ (t = 1, \cdots, 6) \quad (5)
\]