ANSWERS TO ALGEBRA PROBLEMS

Q1, Sec 03. **Grading On The Curve** (**GOTC**), Inc. administers Operations Management tests to job candidates who are applying for CEO positions in leading Fortune 500 Companies. For this purpose, GOTC employs eight senior graders and ten junior graders (called in-house graders)

Each test consists of three parts: written, computer, and video-recorded. The time, in minutes, required by each kind of grader to grade each type of test is as follows:

Grader	Written	Computer	Recorded
Senior	20	15	15
Junior	30	20	18

Each grader can work up to 40 hours in the coming week. 800 candidates have just taken the test, and the results have to be announced after a week. For quality control purposes, senior graders must grade at least 10% of all tests graded in-house in each category.

Work can also be assigned to outside part-time graders, who charge \$25 for each written test, \$15 for each computer test, and \$20 for each recorded test, and can do any number of tests in a week. However, each test done by an outside grader requires a check by an in-house grader. A check by a senior grader takes 10 minutes, while a check by a junior grader takes 15 minutes.

Formulate an algebraic linear programming model to minimize the wages paid to outside graders, while still meeting the deadline for grading all the tests. Clearly define the decision variables, and explain the constraints.

Solution

Variables

For

type of exam X ($X = W, C, R$ for **Written**, **Computer**, and **Recorded**, respectively), and
kind of grader Y ($Y = S, J, P$ for **Senior**, **Junior**, and **Part-time** (outside) **grader**), denote by
XY = the **number of exams of type** X **graded by graders of kind** Y.

For example,

RJ = the number of recorded exams graded by junior graders,

CP = the number of computer exams graded by outside graders.

Also,

ES, EJ the **number of exams checked by senior, and junior graders**, respectively.

Objective

$$\min 25\,WP + 15\,CP + 20\,RP$$

Constraints

$$WS + WJ + WP = 800 \tag{1}$$
$$CS + CJ + CP = 800 \tag{2}$$
$$RS + RJ + RP = 800 \tag{3}$$
$$WP + CP + RP = ES + EJ \tag{4}$$
$$\tfrac{20}{60}\,WS + \tfrac{15}{60}\,CS + \tfrac{15}{60}\,RS + \tfrac{10}{60}\,ES \le 8 \cdot 40 \tag{5}$$
$$\tfrac{30}{60}\,WJ + \tfrac{20}{60}\,CJ + \tfrac{18}{60}\,RJ + \tfrac{15}{60}\,EJ \le 10 \cdot 40 \tag{6}$$
$$WS \ge 0.1(WS + WJ) \tag{7}$$
$$CS \ge 0.1(CS + CJ) \tag{8}$$
$$RS \ge 0.1(RS + RJ) \tag{9}$$
$$\text{All variable} \ge 0$$

Q1, Sec 10. **Manpower Planning**. The **Garden State Airline** (GSA) must decide how many new cabin personnel (stewardesses & stewards) to hire and train over the next 6 months. The requirements are given in terms of flight-hours per month:

Month	Hours Needed	Month	Hours Needed
January	8,000	April	10,000
February	9,000	May	9,000
March	8,000	June	12,000

One month of training is required before a trainee becomes "experienced" (or a "regular crew member"). During the month of training, each trainee requires 100 hours of actual in-flight experience and supervision. Therefore, for each trainee, 100 less hours are available for flight service by regular personnel.

Each experienced crew member can work up to 150 hours in a month. The pay for experienced crew members is \$3000/month, even if they work less than 150 hours. Experienced crew members may be asked to work up to 20 hours overtime, at the rate of \$50/hour (in addition to their regular hours and pay.) The pay for trainees is \$2000/month.

GSA has 60 regular crew members at the beginning of January. Each month afterwards, approximately 10% of the experienced personnel quit GSA on the first of the month.

Formulate the hiring-and-training problem as an LP model, where the objective is to minimize cost. Use X_t for the **number of new personnel beginning training in month** t, $(t = 1, \cdots, 6)$. Define any additional variables that you need.

Solution

Variables

For each month $t = 1, \cdots, 6$:
X_t the **number of new personnel beginning training in month** t,
Y_t the **number of experienced personnel reporting to work at the beginning of month** t,
O_t the **number of hours worked overtime in month** t

Objective

$$\min \left\{ 3,000 \sum_{t=1}^{6} Y_t + 2,000 \sum_{t=1}^{6} X_t + 50 \sum_{t=1}^{6} O_t \right\}$$

Constraints

For each month $t = 1, \cdots, 6$

$$150\,Y_t + O_t - 100\,X_t \geq \text{hours needed in month } t \tag{1}$$

In particular

$$\begin{aligned} 150\,Y_1 + O_1 - 100\,X_1 &\geq 8,000 \\ \cdots \quad & \quad \cdots \\ 150\,Y_6 + O_6 - 100\,X_6 &\geq 12,000 \end{aligned}$$

For each month $t = 1, \cdots, 5$

$$Y_{t+1} = 0.9 Y_t + X_t, \ (t = 1, \cdots, 6) \quad \text{(personnel balance)} \tag{2}$$

$$Y_1 = 60 \quad \text{(initial personnel)} \tag{3}$$

For each month $t = 1, \cdots, 6$

$$O_t \leq 20\,Y_t \quad \text{(limit on overtime)} \tag{4}$$

$$X_t, Y_t, O_t \geq 0, \ (t = 1, \cdots, 6) \tag{5}$$