SOLUTION OF PROBLEM 1

Variables

\( T \) = project duration,
For activity \( X \) \((X = A, \cdots, G)\)
\( t_X \) = the earliest time activity \( X \) can be started,
\( y_X \) = the number of days to crash from \( X \),
\( \delta_X \) = the decision to crash \( X \), i.e.,
\[
\delta_X = \begin{cases} 
1, & \text{if } y_X > 0; \\
0, & \text{otherwise.}
\end{cases}
\]

\( \epsilon \) = the decision to crash \( D \) by more than 4 days, i.e.,
\[
\epsilon = \begin{cases} 
1, & \text{if } y_D > 4; \\
0, & \text{otherwise.}
\end{cases}
\]

\( y_{D1} \) = the number of days to crash from \( D \) at regular variable cost of \( $400/day \),
\( y_{D2} \) = the number of days to crash from \( D \) at discounted variable cost of \( $200/day \).

Auxiliary computation:

Fixed Cost = \( 600\delta_A + 400\delta_B + 500\delta_C + 600\delta_D + 500\delta_E + 700\delta_F + 800\delta_G \)
Variable Cost = \( 200y_A + 500y_B + 300y_C + 400y_{D1} + 200y_{D2} + 200y_E + 400y_F + 200y_G \)
Total Cost = Fixed Cost + Variable Cost.

Part (a):

Objective

\[ \min \text{ Total Cost} \]

Constraints

Deadline

\[ T = 18, \quad (1) \]

Start times

\[ T \geq t_G + (13 - y_G), \quad T \geq t_F + (8 - y_F) \quad (2a) \]
\[ t_G \geq t_D + (9 - y_D), \quad t_G \geq t_E + (10 - y_E) \quad (2b) \]
\[ t_F \geq t_C + (7 - y_C), \quad t_F \geq t_D + (9 - y_D) \quad (2c) \]
\[ t_E \geq t_C + (7 - y_C) \quad (2d) \]
\[ t_D \geq t_A + (8 - y_A), \quad t_D \geq t_B + (12 - y_B) \quad (2e) \]
\[ t_C \geq t_A + (8 - y_A), \quad t_D \geq t_B + (12 - y_B) \quad (2f) \]
\[ t_A \geq 0, \quad t_B \geq 0 \quad (2g) \]
Crashes

\[ 2 \delta_A \leq y_A \leq 5 \delta_A \]  \hspace{1cm} (3a)
\[ 3 \delta_B \leq y_B \leq 3 \delta_B \]  \hspace{1cm} (3b)
\[ \delta_C \leq y_C \leq 4 \delta_C \]  \hspace{1cm} (3c)
\[ y_D = y_{D1} + y_{D2} \quad \text{(total days crashed from } D) \]  \hspace{1cm} (3d)
\[ 2 \delta_D \leq y_{D1} \leq 4 \delta_D \quad \text{(regular cost between 2 and 4 days)} \]  \hspace{1cm} (3e)
\[ y_{D1} \geq 4 \epsilon \quad \text{(must have 4 days at regular cost, before getting discount)} \]  \hspace{1cm} (3f)
\[ 0 \leq y_{D2} \leq 3 \epsilon \quad \text{(can have discounted cost at most 3 days)} \]  \hspace{1cm} (3g)
\[ 4 \delta_E \leq y_E \leq 6 \delta_E \]  \hspace{1cm} (3h)
\[ 3 \delta_F \leq y_F \leq 5 \delta_F \]  \hspace{1cm} (3i)
\[ y_F \geq y_C \quad \text{(days crashed from } F \text{ no less than days crashed from } C) \]  \hspace{1cm} (3j)
\[ 5 \delta_G \leq y_G \leq 9 \delta_G \]  \hspace{1cm} (3k)

Logical conditions

\[ \delta_A + \delta_B \leq 1 \quad \text{(at most one of } A, B) \]  \hspace{1cm} (4a)
\[ \epsilon \leq \delta_D \quad \text{(can’t have discount without first paying regular)} \]  \hspace{1cm} (4b)

Conditions on variables

\[ \delta_A, \ldots, \delta_G, \epsilon = \text{binary} \]  \hspace{1cm} (5)

\[ \text{(the start times } y_X \text{ are } \geq 0 \text{ by (3a)-(3k))} \]

Part (b):

**Objective**

\[ \min T \]

**Constraints**

**Budget**

\[ \text{Total Cost } \leq 8,000 \]  \hspace{1cm} (6)

and all constraints of Part (a) except (1).