SOLUTION OF PROBLEM 1

Variables

T =**project duration**,
For **activity** X $(X = A, \cdots, G)$
t_X = the **earliest time activity** X **can be started**,
y_X = the **number of days to crash from** X,
δ_X = the **decision to crash** X, i.e.,

$$\delta_X = \begin{cases} 1, & \text{if } y_X > 0; \\ 0, & \text{otherwise.} \end{cases}$$

ϵ = the **decision to crash** D **by more than 4 days**, i.e.,

$$\epsilon = \begin{cases} 1, & \text{if } y_D > 4; \\ 0, & \text{otherwise.} \end{cases}$$

y_{D1} = the **number of days to crash from** D **at regular variable cost of** \$400/day,
y_{D2} = the **number of days to crash from** D **at discounted variable cost of** \$200/day.

Auxiliary computation:

$$\begin{aligned} \textbf{Fixed Cost} &= 600\,\delta_A + 400\,\delta_B + 500\,\delta_C + 600\,\delta_D + 500\,\delta_E + 700\,\delta_F + 800\,\delta_G \\ \textbf{Variable Cost} &= 200\,y_A + 500\,y_B + 300\,y_C + 400\,y_{D1} + 200\,y_{D2} + 200\,y_E + 400\,y_F + 200\,y_G \\ \textbf{Total Cost} &= \text{Fixed Cost} + \text{Variable Cost}\ . \end{aligned}$$

Part (a):

Objective

$$\min \textbf{ Total Cost}$$

Constraints

Deadline

$$T = 18\ , \tag{1}$$

Start times

$$T \geq t_G + (13 - y_G)\ ,\ T \geq t_F + (8 - y_F) \tag{2a}$$
$$t_G \geq t_D + (9 - y_D)\ ,\ t_G \geq t_E + (10 - y_E) \tag{2b}$$
$$t_F \geq t_C + (7 - y_C)\ ,\ t_F \geq t_D + (9 - y_D) \tag{2c}$$
$$t_E \geq t_C + (7 - y_C) \tag{2d}$$
$$t_D \geq t_A + (8 - y_A)\ ,\ t_D \geq t_B + (12 - y_B) \tag{2e}$$
$$t_C \geq t_A + (8 - y_A)\ ,\ t_D \geq t_B + (12 - y_B) \tag{2f}$$
$$t_A \geq 0\ ,\ t_B \geq 0 \tag{2g}$$

Crashes

$$2\,\delta_A \leq y_A \leq 5\,\delta_A \tag{3a}$$

$$3\,\delta_B \leq y_B \leq 3\,\delta_B \tag{3b}$$

$$\delta_C \leq y_C \leq 4\,\delta_C \tag{3c}$$

$$y_D = y_{D1} + y_{D2} \quad \text{(total days crashed from } D) \tag{3d}$$

$$2\,\delta_D \leq y_{D1} \leq 4\,\delta_D \quad \text{(regular cost between 2 and 4 days)} \tag{3e}$$

$$y_{D1} \geq 4\,\epsilon \quad \text{(must have 4 days at regular cost, before getting discount)} \tag{3f}$$

$$0 \leq y_{D2} \leq 3\,\epsilon \quad \text{(can have discounted cost at most 3 days)} \tag{3g}$$

$$4\,\delta_E \leq y_E \leq 6\,\delta_E \tag{3h}$$

$$3\,\delta_F \leq y_F \leq 5\,\delta_F \tag{3i}$$

$$y_F \geq y_C \quad \text{(days crashed from } F \text{ no less than days crashed from } C) \tag{3j}$$

$$5\,\delta_G \leq y_G \leq 9\,\delta_G \tag{3k}$$

Logical conditions

$$\delta_A + \delta_B \leq 1 \quad \text{(at most one of } A,\ B) \tag{4a}$$

$$\epsilon \leq \delta_D \quad \text{(can't have discount without first paying regular)} \tag{4b}$$

Conditions on variables

$$\delta_A, \cdots, \delta_G, \epsilon = \text{binary} \tag{5}$$

(the start times y_X are ≥ 0 by (3a)–(3k))

Part (b):

Objective

$$\min T$$

Constraints

Budget

$$\text{Total Cost} \leq 8,000 \tag{6}$$

and all constraints of Part (a) except (1).